Tessellation cover for bounded treewidth graphs

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Introduction

A *tessellation* of a graph *G* is a partition of its vertex set V(G) into vertex disjoint cliques. A *tessellation cover* of G is a set of tessellations that covers its edge set E(G). The *tessellation cover number* T(G) of a graph G is the size of a minimum tessellation cover of G. The *k*-TESSELABILITY problem aims to decide whether T(G) ≤ k for a graph G and an positive integer k.

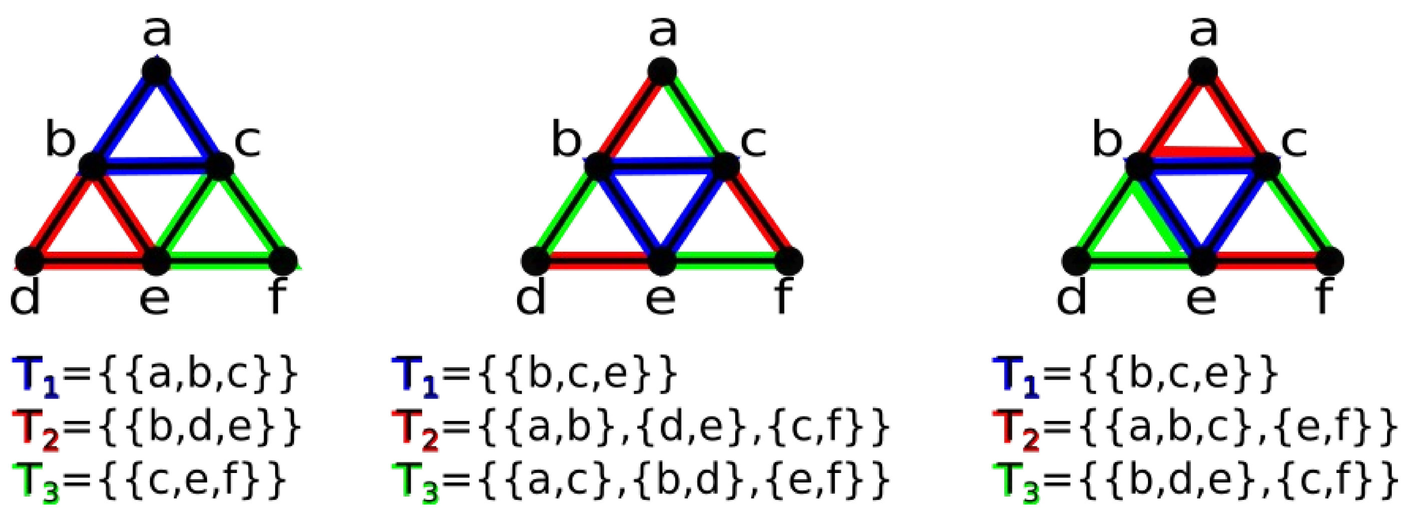

Figure 1: Three tessellation covers of the Hajós' graph using three tessellations..

A graph *G* is a *q-tree* if G is clique of size *q* or if *G* is obtained from a *q-tree* by adding a new vertex adjacent to a clique of size *q*. A graph *G* is a *partial q-tree* if it is a subgraph of a *q-tree*. The *treewidth* of a graph *G* is the smallest *q* such that *G* is a partial q-tree.

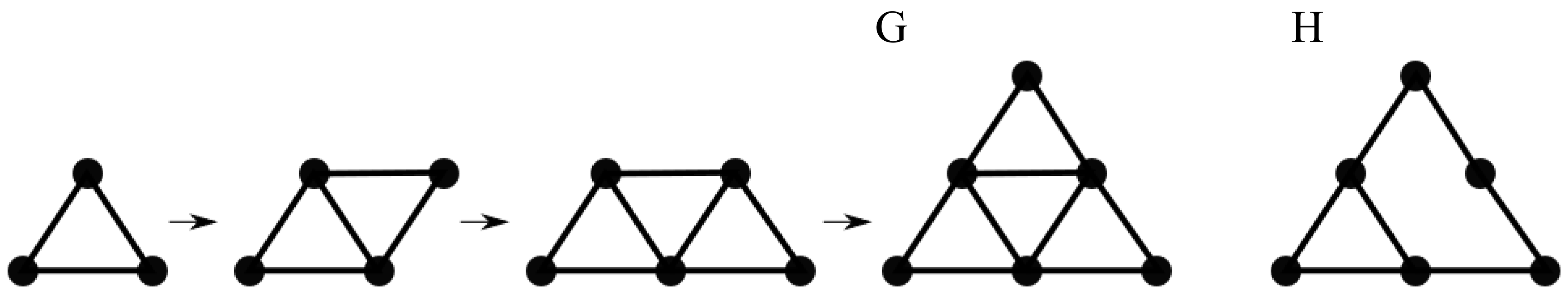

Figure 2: The 3-tree construction of the Hajós' graph G and a subgraph H of G that has treewidth 3.

In this work we show that the *k*-TESSELABILITY problem can be described in monadic second order logic 2 MSOL2. As a consequence, we obtain a linear time algorithm to solve *k*-TESSELABILITY when *k* is fixed for bounded treewidth graphs.

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Motivations and Related Works

Portugal et al. [6] introduced the concept of tessellation cover on graphs. They also established a modern application for tessellations by showing a close relation with quantum walk models. Other applications, such for frequency assignment problem, were described in [4].

There are several works related to the computational complexity of *k*-TESSELABILITY problem [1, 2, 3, 7]. We highlight that the problem is NP-complete for fixed values of k ≥ 3, chordal graphs, planar graphs, bipartite graphs, (1,2)-graphs, (2,1)-graphs, line graph of triangle-free graphs and universal graphs[]. Moreover, there are polynomial time-algorithms to solve it when k=2, for diamond-free clique perfect graphs, and good tessellable graphs.

Results

The Courcelle's Theorem states that any graph theory problem described in monadic second-order logic (MSOL) can be decided in linear time on bounded treewidth graphs [4]. Courcelle's Theorem can also be used with a variation of monadic second-order logic called MSOL2, which allows quantification over the set of edges [5].

We describe the k-TESSELABILITY problem for a fixed k with MSOL2 restricting that each edge of the graph must belong to at least one of the k tessellations and that the group of edges of a tessellation must respect the following restriction: if the edges uv and vw belong to a tessellation, then the edge uw must belong to it.

The first restriction can be described as: $\forall uv \in E(G),\ \exists\, t \in T,\ L(u, v, t)$
where *T* is a set with *k* tessellations and *L(u, v, c)* is true when uv belong to tessellation *t*.

The second restriction can be described as:
$\forall uv \in E(G), \forall vw \in E(G), (\ L(u, v, t)\ \wedge\ L(v, w, t) \wedge u \neq w) \ \rightarrow\ (E(u,w) \wedge L(u, w, t)))$
where E(u,w) is true when the edge uw ∈ E(G).

Ongoing works

We are currently studying the tessellation cover problem for other known graph theory parameters such as cliquewidth and pathwidth.

References

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