Flood-it Game on Co-comparability Graphs

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A flooding game is a game on a pre-colored board where each cell has a color, and in each turn, the player assigns a new color to one of the cells on the board. When a cell receives a new color, it is merged with all its neighbors with the same color. The goal of the game is to obtain a monochromatic board using the minimum number of turns. When the board cells are squares, the game is called Flood-it, and when they are hexagons, the game is called HoneyBee or Mad Virus. There are two versions of these games: in the free-pivot version, the player can choose in each turn what cell he wants to color; in the fixed-pivot version, it is always the same cell that receives a new color.

The flooding games can be generalized for graphs: given a graph whose vertices were previously colored, a new color is assigned to the pivot in each turn and this assignment of color is propagated to every vertex connected to the pivot by a monochromatic path (considering the coloring as when the turn starts). As in the board version, the goal is to obtain a graph whose vertices are all colored the same, using the minimum number of turns.

The fixed-pivot version of the Flood-it game is polynomial-time solvable for interval graphs (Fukui, et. al, On Complexity of Flooding Games on Graphs with Interval Representations, 2012), for the powers of paths P_n^2 and the powers of cycles C_n^2 (Souza, et. al, An algorithmic analysis of Flood-It and Free-Flood-It on graph powers, 2014), and for $2 \times n$ circular grid graphs (Souza, et. al, An algorithmic analysis of Flood-It and Free-Flood-It on graph powers, 2014). However, the free version of the Flood-it game is NP-Complete for interval graphs (Fukui, et. al, On Complexity of Flooding Games on Graphs with Interval Representations, 2012), and it is NP-hard for powers of cycles C_n^2 and $3 \times n$ circular grid graphs (Souza, et. al, An algorithmic analysis of Flood-It and Free-Flood-It on graph powers, 2014).

A co-comparability graph is the intersection graph of curves between two parallel lines. Fleischer and Woeginger (An algorithmic analysis of the Honey-Bee game, 2012) proved that the fixed-pivot Flood-it game is polynomial-time solvable for co-comparability graphs. They present two algorithms to solve that problem: a single source shortest path algorithm and a dynamic programming. In this note, we claim that the recursive definition of their dynamic programming is not well-defined, and we present an example for which it does not finish.

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