Tessellation cover for bounded treewidth graphs

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Keywords: Graph Theory, Graph Coloring, Computational Complexity

A *tessellation* of a graph G is a partition of its vertex set $V(G)$ into vertex disjoint cliques. A *tessellation cover* of G is a set of tessellations that covers its edge set $E(G)$. The *tessellation cover number* $T(G)$ of a graph G is the size of a minimum tessellation cover of G. The k-TESSELABILITY problem aims to decide whether $T(G) \leq k$ for a graph G and an positive integer k. This problem is $\mathcal{NP}$-complete for any fixed $k \geq 3$, chordal graphs or planar graphs (Abreu et al., The graph tessellation cover number: chromatic bounds, efficient algorithms and hardness, *Theoretical Computer Science* 801, 175-191, 2020). A graph G is a *q-tree* if G is clique of size q or if G is obtained from a q-tree by adding a new vertex adjacent to a clique of size q. A graph G is a *partial q-tree* if it is a subgraph of a q-tree. The *treewidth* of a graph G is the smallest q such that G is a partial q-tree.

The Courcelle's Theorem states that any graph theory problem described in monadic second-order logic (MSOL) can be decided in linear time on bounded treewidth graphs (Courcelle, The monadic second–order theory of graphs I: Recognizable sets of finite graphs, INFORMATION AND COMPUTATION 85, 12–75, 1990). Courcelle's Theorem can also be used with a variation of monadic second-order logic called MSOL2, which allows quantification over the set of edges (cf. Pires et al., Revisando o Teorema de Courcelle, PROCEEDINGS OF LI SBPO, v.2, 107372, 2019). In this work we show that the k-TESSELABILITY problem can be described in monadic second order logic 2 ($MSOL_2$). As a consequence, we obtain a linear time algorithm to solve k-TESSELABILITY when k is fixed for bounded treewidth graphs.

ISSN:2526-1371 DOI:10.5281/zenodo.6090322