Workshop de Pesquisa em Computação dos Campos Gerais 2021

Flood-it Game on Co-comparability Graphs

Guilherme Quintero Lorenzi[1,2] e Sheila Morais de Almeida[1,3]

UNIVERSIDADE TECNOLÓGICA FEDERAL DO PARANÁ
CÂMPUS PONTA GROSSA

Flood-it Game

The *Flood-it game* is a game on a pre-colored board where each cell has a color, and in each turn, the player assigns a new color to one of the cells on the board. When a cell receives a new color, it is merged with all its neighbors with the same color (see Figure 1). The goal of the game is to obtain a monochromatic board using the minimum number of turns. There are two versions of this game: the free-pivot version and the fixed-pivot version.

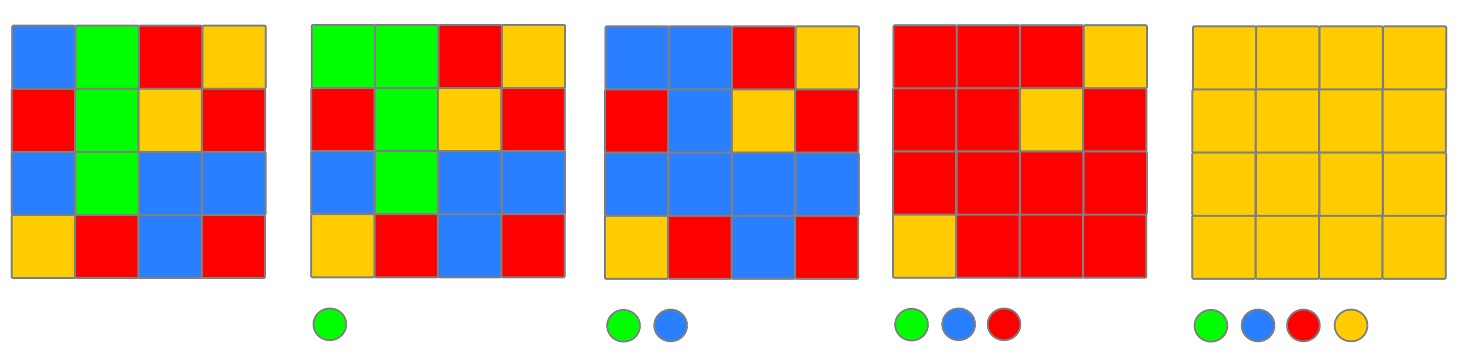

Figure 1: Flood-it game example

The Flood-it game can be generalized for graphs: given a graph whose vertices were previously colored, a new color is assigned to the pivot in each turn and this assignment of color is propagated to every vertex connected to the pivot by a monochromatic path (considering the coloring as when the turn starts).

Previous Results

The fixed pivot version of the Flood-it game is polynomial-time solvable for interval graphs (Hiroyuki et al., 2011), powers of paths P_n^2 and power of cycles C_n^2 (Souza et al., 2014), and $2 \times n$ circular grid graphs (Souza et al., 2014).

Co-comparability Graphs

A *co-comparability graph* is the intersection graph of curves between two parallel lines (see Figure 2). Fleischer and Woeginger (2012) proved that the fixed-pivot Flood-it game is polynomial-time solvable for co-comparability graphs. Figure 2 presents a flooding sequence on a co-comparability graph where v_0 is the fixed pivot.

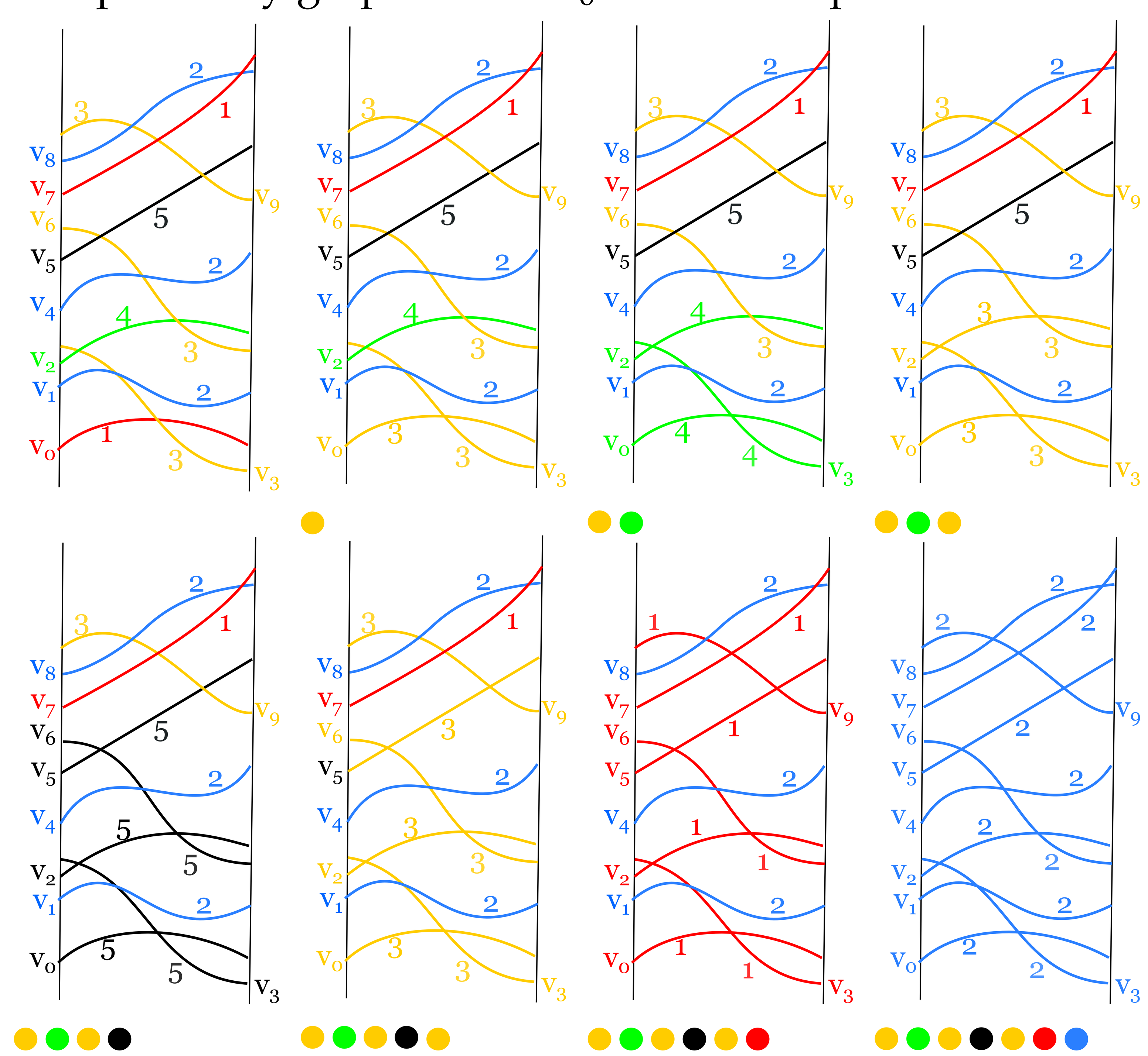

Figure 2: Flood-it game on a co-comparability graph

Fleischer and Woeginger (2012) present two algorithms to solve the Flood-it game on co-comparability graphs: a single source shortest path algorithm and a dynamic programming, defined as follows.

- $Max(c)$: the largest node of color c;
- M: the set of nodes $Max(c)$, for all colors c;
- M_{max}: the set of maximal elements of the partial order;
- $ess(\gamma)$: $|\gamma|$ minus the number of steps where γ conquers a node in M.
- $D(v) = ess(\gamma)$, where γ is a shortest color sequence that conquers v when starting at v_0.

The optimal solution is the $\min_{v \in M_{max}}(D(v)) + k$, where k is the number of colors in the original graph. The value of $D(v)$ can be defined recursively:

$$D(v_i) = \begin{cases} 0, \text{ if } i = 0 \ (v_0 \text{ is the pivot}); \\ \min_c(D(min_{v_i}(c) + \delta(v_i)), \text{if } i > 0, \end{cases}$$

where $\delta(v_i) = 0$, if $v \in M$; and $\delta(v_i) = 1$ otherwise.

Result

We claim that this recursive definition of $D(V)$ is not well-defined. We present the following counterexample.

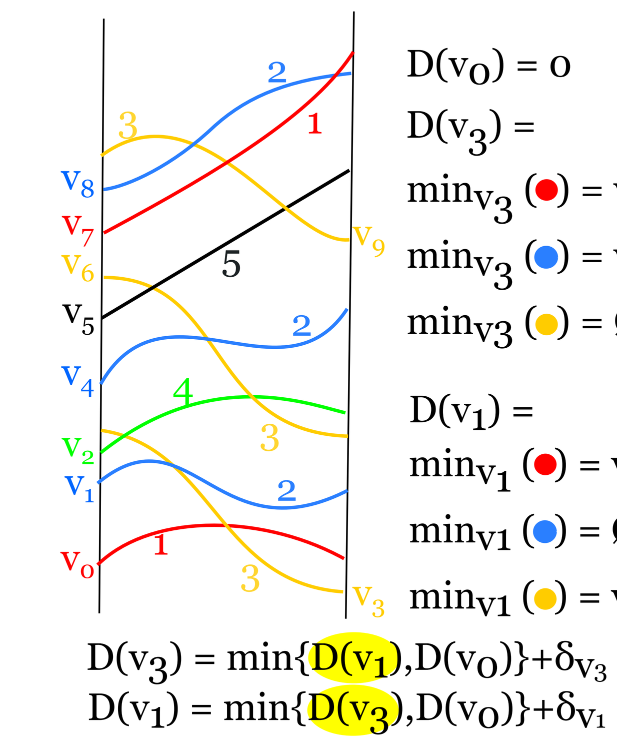

Figure 3: Counterexample for the dynamic programming of Fleischer and Woeginger (2012)

References

Fleischer, Rudolf, and Gerhard J Woeginger. 2012. An algorithmic analysis of the honey-bee game. *Theoretical Computer Science* 452:75–87.

Hiroyuki, Fukui, Nakanishi Akihiro, Uehara Ryuhei, Uno Takeaki, and Uno Yushi. 2011. The complexity of free flood filling games. *Proceedings of The 14th Korea-Japan Joint Workshop on Algorithms and Computation* 2011(7):1–5.

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[1]Universidade Tecnológica Federal do Paraná - Câmpus Ponta Grossa, Departamento Acadêmico de Informática
[2]guilhermequintero@alunos.utfpr.edu.br; [3]sheilaalmeida@utfpr.edu.br