Equitable Total Coloring of Small Graphs

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A total k-coloring of a simple graph G = (V, E) assigns at most k colors to the vertices and edges of G such that distinct colors are assigned to every pair of adjacent vertices in V, to every pair of adjacent edges in E, and each vertex and its incident edges. A total coloring is equitable if the cardinalities of any two color classes differ by at most 1. The total chromatic number $\chi''(G)$ is the minimum k such that G admits a total k-coloring. Behzad and Vizing independently conjectured that for every simple graph, $\Delta + 1 \leq \chi''(G) \leq \Delta + 2$, know as the Total Coloring Conjecture, where Δ is the maximum degree in G. Graphs that satisfy $\chi''(G) = \Delta + 1$ are called Type 1.

In 2020, Stemock considered equitable total colorings of cubic graphs in his article "On the equitable total (k + 1)-coloring of k-regular graphs". The author conjectured that every total 4-coloring of a cubic graph is equitable if n < 20. This upper bound was motivated by a graph with n = 20 in the article "On the equitable total chromatic number of cubic graphs", by Dantas, Figueiredo, Mazzuoccolo, Preissmann, dos Santos and Sasaki, that is Type 1 but does not have an equitable total coloring with 4 colors. This conjecture becomes relevant when we realize that it refers to more than 40000 graphs, which can be verified in the website "House of Graphs". We found counter examples for the conjecture with n = 12 and 18 in the article "Some refinements of the total chromatic number conjecture", by Chetwynd and Hilton, where non equitable total colorings of these Type 1 graphs are described.

So far all small Type 1 cubic graphs we have analyzed have at least one equitable total 4-coloring. We propose instead the following conjecture: every Type 1 cubic graph has at least one equitable total 4-coloring for n < 20.